B.Sc Physics Part-I

Paper-I, Group B.

Hamilton's equations:

In earlier lecture note we have introduced the Hamiltonian function H. In terms of generalized coordinate and momenta, we write H

$$H = H(q_1, q_2, q_3, \cdots q_k, \cdots q_n;\ p_1, p_2, p_3, \cdots p_k, \cdots p_n;\ t) \quad \text{--- (1)}$$

The differential of H is given by

$$dH = \sum_k \frac{\partial H}{\partial q_k} dq_k + \sum_k \frac{\partial H}{\partial p_k} dp_k + \frac{\partial H}{\partial t} dt \quad \text{--- (2)}$$

Since we know $\quad H = \sum_k p_k \dot{q}_k - L \quad$ --- (3)

From (3)

$$dH = \sum_k \dot{q}_k dp_k + \sum_k p_k d\dot{q}_k - dL \quad \text{--- (4)}$$

$$dL = \sum_k \frac{\partial L}{\partial q_k} dq_k + \sum_k \frac{\partial L}{\partial \dot{q}_k} d\dot{q}_k + \frac{\partial L}{\partial t} dt$$

using $\dot{p}_k = \frac{\partial L}{\partial q_k}$ and $p_k = \frac{\partial L}{\partial \dot{q}_k}$, we write

$$dL = \sum_k \dot{p}_k dq_k + \sum_k p_k d\dot{q}_k + \frac{\partial L}{\partial t} dt \quad \text{--- (5)}$$

Now using Eq. (5) in Eq. (4)

$$dH = \sum_k \dot{q}_k dp_k + \cancel{\sum_k p_k d\dot{q}_k} - \sum_k \dot{p}_k dq_k - \cancel{\sum_k p_k d\dot{q}_k} - \frac{\partial L}{\partial t} dt$$

or $$dH = \sum_k \dot{q}_k dp_k - \sum_k \dot{p}_k dq_k - \frac{\partial L}{\partial t} dt \quad \text{--- (6)}$$

Now comparing equations (5) and (6), we obtain.

$$\dot{q}_k = \frac{\partial H}{\partial p_k} \quad \text{— (7)}$$

$$-\dot{p}_k = \frac{\partial H}{\partial q_k} \quad \text{— (8)}$$

and $$\frac{-\partial L}{\partial t} = \frac{\partial H}{\partial t} \quad \text{— (9)}$$

Equations (7) and (8) are Hamilton's equations of motion, also called Canonical equations of motion. Motion described by equations (7) & (8) is called Hamilton's dynamics.

→ Hamilton's equations are first-order differential equations and are easier to solve than the 2nd-order differential equations of Lagrangian dynamics.

H.W. Show that if H does not contain time explicitly, then H is a constant of motion.